C1 JAN 2005

1a) $16^{1/2} = \sqrt{16} = 4$

b) $16^{-3/2} = \frac{1}{16^{3/2}} = \frac{1}{(16^{1/2})^3} = \frac{1}{(\sqrt{16})^3} = \frac{1}{4^3} = \frac{1}{64}$

2i) $y = 5x^3 + 7x + 3$

a) $\frac{dy}{dx} = 15x^2 + 7$

b) $\frac{d^2y}{dx^2} = 30x$

ii) $\int \left(1 + 3\sqrt{x} - \frac{1}{x^2} \right) dx$

$= \int 1 + 3x^{1/2} - 1x^{-2} \, dx$

$= x + \frac{6}{3}x^{3/2} + x^{-1} + \underline{\underline{c}}$

$= x + 2x^{3/2} + \frac{1}{x} + c$

3) $kx^2 + 12x + k = 0$

divide by k $\quad x^2 + \frac{12}{k}x + \underline{\underline{1}} = 0$

can only be from 1x1

roots are equal = same

roots $= (x+1)^2$

$(x+1)(x+1) = x^2 + 2x + 1$

equate coeffs of x

$2 = \frac{12}{k}$

$k = \frac{12}{2} = 6 \qquad \underline{\underline{k = 6}}$

4) $x+y=2 \quad \ldots (1)$

$x^2+2y=12 \quad \ldots (2)$

from (1) $y=2-x$ sub into (2)

$x^2+2(2-x)=12$
$x^2+4-2x=12$
$x^2-2x-8=0$
$(x-4)(x+2)=0$
$x=4$ or $x=-2$

when $x=4$
$y=2-(4)=-2$

when $x=-2$
$y=2-(-2)=4$

	1st	2nd	3rd		r^{th}
5)	a	$a+d$	$a+2d$	$\ldots$	$a+(r-1)d$

$a+(r-1)d=2r-5$
$a+dr-d=2r-5$
equate coeffs of r
$d=2$
$a+2r-2=2r-5$
first: $\underline{\underline{a=-3}}$

second $=a+d=-3+2$
$\underline{\underline{=-1}}$

$=\boxed{-3,-1,1}$

third $=a+2d=-3+4$
$\underline{\underline{=1}}$

b) common difference $= d = \underline{\underline{2}}$

c) $\sum_{r=1}^{n} (2r-5) = n(n-4)$

$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

$a = -3 \quad d = 2$

$S_n = \frac{1}{2}n\left[-6 + (n-1)2\right]$

$= \frac{1}{2}n\left[-6 + 2n - 2\right]$

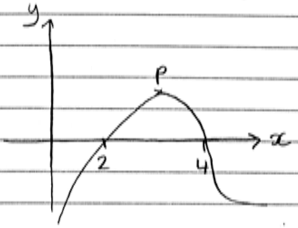
$= \frac{1}{2}n\left[-8 + 2n\right]$

$= -4n + n^2$

$= n(n-4)$

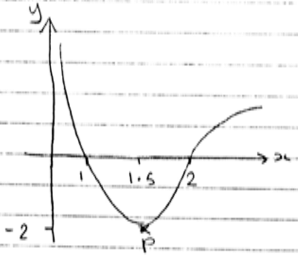
6) a) $y = -f(x) \Rightarrow$ (all y coords $\times -1$) /reflection in x-axi

$(2,0)\,(4,0)\,(3,2)$



b) $y = f(2x) \Rightarrow$ all x coords $\times \frac{1}{2}$

$(1,0)\ (2,0)\ (1\cdot5, -2)$



$y = 4x^2 + \frac{5-x}{x}$

$y = 4x^2 + \frac{5}{x} - \frac{x}{x}$

$y = 4x^2 + 5x^{-1} - 1$

$\frac{dy}{dx} = 8x - 5x^{-2}$

when $x = 1$ at p

$\frac{dy}{dx} = 8(1) - 5(1)^{-2} = 8 - 5 = \underline{\underline{3}}$

) when $x = 1$

$y = 4(1)^2 + \frac{5-(1)}{(1)} = 4 + 4 = 8$

$y - y_1 = m(x - x_1)$

$y - 8 = 3(x - 1)$

$y = 3x - 3 + 8$

$y = 3x + 5$

c)

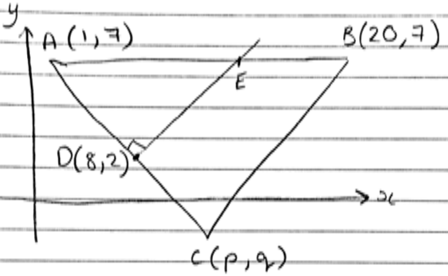
$y = 3x + 5$ at x-axis $y = 0$

$3x + 5 = 0$

$3x = -5$

$x = -\dfrac{5}{3}$

8)



midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$\dfrac{1+p}{2} = 8$ $\qquad$ $\dfrac{7+q}{2} = 2$

$1 + p = 16$ $\qquad$ $7 + q = 4$

$\underline{\underline{p = 15}}$ $\qquad$ $\underline{\underline{q = -3}}$

b) ~~C (15, -3)~~ $\qquad$ m of $\overrightarrow{AD} = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{7-2}{1-8} = \dfrac{5}{-7}$

m of $l = \dfrac{7}{5}$ as perpendicular

$y - y_1 = m(x - x_1)$ at D (8,2)

$y - 2 = \dfrac{7}{5}(x - 8) \Rightarrow 5y - 10 = 7x - 56$

$5y = 7x - 46$

$-7x + 5y + 46 = 0$

c) line AB = $y=7$

sub in to

$-7x+5y+46=0$ $\quad$ 35 46 81

$-7x+5(7)+46=0$

$-7x+35+46=0$

$-7x+81=0$

$-7x=-81$

$x=\frac{-81}{-7}$ $\quad$ $\boxed{x=\frac{81}{7}}$

)

md of C = $\frac{dy}{dx}=(3x-1)^2$

at point (1,4)

$\frac{dy}{dx}=(3(1)-1)^2 = 2^2 = 4$

$\frac{dy}{dx}$ at normal = $-\frac{1}{4}$

$y-y_1=m(x-x_1)$

$y-4=-\frac{1}{4}(x-1)$

$4y-16=-x+1$

$4y=-x+17$

b) $\frac{dy}{dx}=(3x-1)^2 = (3x-1)(3x-1) = 9x^2-6x+1$

$\int\frac{dy}{dx} = y = \int 9x^2-6x+1 \, dx$

$y=\frac{9}{3}x^3-\frac{6}{2}x^2+x+c$

point P(1,4) is on C so when $x=1$ $y=4$

$y = \frac{9}{3}x^3 - \frac{6}{2}x^2 + x + c$

$y = 3x^3 - 3x^2 + x + c$

$4 = 3(1)^3 - 3(1)^2 + 1 + c$

$4 = 3 - 3 + 1 + c$

$\underline{\underline{c = 3}}$

$y = 3x^3 - 3x^2 + x + 3$

c) parallel to $y = 1 - 2x$
means same gradient $= -2$

squared always ≥ 0

tangent to C gradient $= (3x-1)^2 \geq 0$

$-2 < 0$ ∴ cannot have same gradient ∴ cannot be parallel.

10) $f(x) = x^2 - 6x + 18$

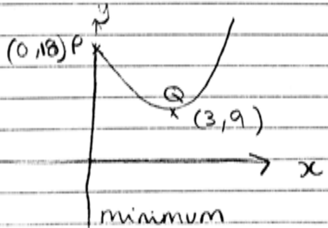
complete the square

$f(x) = \left(x - \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 18$

$= (x-3)^2 - 9 + 18$

$= (x-3)^2 + 9$

b)



meet y-axis at $x = 0$

$y = 18$

minimum

$(x-3)^2 = 0$

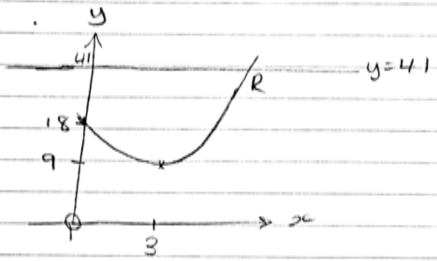
$x = 3$

$y = 3^2 - 6(3) + 18$

$y = 9 - 18 + 18 = 9$

minimum $= (3, 9)$

c)



$41 = x^2 - 6x + 18$

$0 = x^2 - 6x - 23$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{6 \pm \sqrt{36 - 4 \times -23}}{2}$

$23 \times 4 = 92$

$\dfrac{6 \pm \sqrt{36 + 92}}{2}$

$36 + 92 = 128$

$\dfrac{6 \pm \sqrt{128}}{2}$

$+6$

$128 \div 4 = 032$

$\dfrac{6 \pm \sqrt{4} \times \sqrt{32}}{2}$

$= \dfrac{6 \pm \sqrt{4} \times \sqrt{16} \times \sqrt{2}}{2} = \dfrac{6 \pm 2 \times 4\sqrt{2}}{2}$

$3 \pm 4\sqrt{2}$

$x > 0$

$\therefore R = 3 + 4\sqrt{2}$